TRIGONOMETRIA

LISTA DE REVISÃO

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Exercícios

1) Determine o domínio, o conjunto imagem e o período das funções trigonométricas abaixo:

$$a)\ f(x) = 3 + 4.sen\left(2x - \frac{\pi}{4}\right)$$

$$\boldsymbol{Domínio = R}$$

$$\begin{cases} \boldsymbol{sen\left(2x - \frac{\pi}{4}\right) = 1 \rightarrow 3 + 4.1 = 7} \\ \boldsymbol{sen\left(2x - \frac{\pi}{4}\right) = -1 \rightarrow 3 + 4.(-1) = -1} \end{cases} \qquad \boldsymbol{Im = [-1, 7]}$$

$$\boldsymbol{p = \frac{2\pi}{k} \rightarrow p = \frac{2\pi}{2} \rightarrow p = \pi}$$

$$b)\ y = 5 - 2.cos\left(\frac{3x}{2}\right)$$

$$\boldsymbol{Domínio = R}$$

$$\begin{cases} cos\left(\frac{3x}{2}\right) = 1 \rightarrow 5 - 2.1 = 3 \\ cos\left(\frac{3x}{2}\right) = -1 \rightarrow 5 - 2.(-1) = 7 \end{cases}$$

$$\boldsymbol{Im = [3, 7]}$$

$$p = \frac{2\pi}{k} \rightarrow p = \frac{2\pi}{\frac{3}{2}} \rightarrow p = 2\pi.\frac{2}{3} \rightarrow p = \frac{4\pi}{3}$$

$c)\ f(x) = 3.sen\left(\frac{x}{2}\right)$

$\boldsymbol{Domínio = R}$

$$\begin{cases} sen\left(\frac{x}{2}\right) = 1 \rightarrow 3.1 = 3 \\ sen\left(\frac{x}{2}\right) = -1 \rightarrow 3.(-1) = -3 \end{cases}$$

$$Im = [-3, 3]$$

$$p = \frac{2\pi}{k} \rightarrow p = \frac{2\pi}{\frac{1}{2}} \rightarrow p = 2\pi.2 \rightarrow p = 4\pi$$

$d)\ y = 1 - cos\left(\frac{4x}{5} - \pi\right)$

$\boldsymbol{Domínio = R}$

$$\begin{cases} cos\left(\frac{4x}{5} - \pi\right) = 1 \rightarrow 1 - 1 = 0 \\ cos\left(\frac{4x}{5} - \pi\right) = -1 \rightarrow 1 - (-1) = 2 \end{cases}$$

$$Im = [0, 2]$$

$$p = \frac{2\pi}{k} \rightarrow p = \frac{2\pi}{\frac{4}{5}} \rightarrow p = 2\pi.\frac{5}{4} \rightarrow p = \frac{10\pi}{4} \rightarrow p = \frac{5\pi}{2}$$

$e)\ y = 2 + tg\left(\frac{x}{2} + \pi\right)$

$\boldsymbol{Domínio \rightarrow \left(\frac{x}{2} + \pi\right) \neq \frac{\pi}{2} + k\pi \rightarrow \frac{x}{2} + \frac{2\pi}{2} \neq \frac{\pi}{2} + \frac{2k\pi}{2} \rightarrow x + 2\pi \neq \pi + 2k\pi}$

$\boldsymbol{x \neq -\pi + 2k\pi \rightarrow Dom. = \{x \in R | x \neq -\pi + 2k\pi, k \in Z\}}$

$\boldsymbol{Conjunto\ Imagem = R}$

$\boldsymbol{p = \frac{\pi}{k} \rightarrow p = \frac{\pi}{\frac{1}{2}} \rightarrow p = 2\pi}$

$f)\ f(x) = \mathrm{tg}(2x)$

$$\boldsymbol{Domínio \rightarrow 2x \neq \frac{\pi}{2} + k\pi \rightarrow x \neq \frac{\pi}{4} + \frac{k\pi}{2}}$$

$$\boldsymbol{Dom. = \left\{ x \in R \middle| x \neq \frac{\pi}{4} + \frac{k\pi}{2}, k \in Z \right\}}$$

$$\boldsymbol{Conjunto\ Imagem = R}$$

$$\boldsymbol{p = \frac{\pi}{k} \rightarrow p = \frac{\pi}{2}}$$

2) Resolva as expressões abaixo:

$$a)\ E = \frac{sen120° + cos330°}{tg225°}$$

$$sen120° = +sen(180° - 120°) = +sen60° = +\frac{\sqrt{3}}{2}$$

$$cos330° = +cos(360° - 330°) = +cos30° = +\frac{\sqrt{3}}{2}$$

$$tg225° = +tg(225° - 180°) = +tg45° = +1$$

$$E = \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1} \rightarrow E = \frac{2\sqrt{3}}{2} \rightarrow E = \sqrt{3}$$

$$b)\ E=\frac{sec240°-cossec150°}{cotg135°}$$

$$\boldsymbol{sec240°=\frac{1}{cos240°}=\frac{1}{-cos(240°-180°)}=-\frac{1}{cos60°}=-\frac{1}{\frac{1}{2}}=-2}$$

$$\boldsymbol{cossec150°=\frac{1}{sen150°}=\frac{1}{+sen(180°-150°)}=\frac{1}{sen30°}=\frac{1}{\frac{1}{2}}=2}$$

$$\boldsymbol{cotg135°=\frac{1}{tg135°}=\frac{1}{-tg(180°-135°)}=-\frac{1}{tg45°}=-\frac{1}{1}=-1}$$

$$\boldsymbol{E=\frac{-2-(2)}{-1}\rightarrow E=\frac{-4}{-1}\rightarrow E=4}$$

$$c)\ E = cossec\frac{7\pi}{6} + sec\frac{11\pi}{6} + cotg\frac{4\pi}{3} + tg\frac{5\pi}{4}$$

$$\frac{7\pi}{6} = \frac{7.180°}{6} = 7.30° = 210°$$

$$\frac{11\pi}{6} = \frac{11.180°}{6} = 11.30° = 330°$$

$$\frac{4\pi}{3} = \frac{4.180°}{3} = 4.60° = 240°$$

$$\frac{5\pi}{4} = \frac{5.180°}{4} = 5.45° = 225°$$

$$cossec\frac{7\pi}{6} = cossec210° = \frac{1}{sen210°} = \frac{1}{-sen(210° - 180°)} = -\frac{1}{sen30°} = -\frac{1}{\frac{1}{2}} = -2$$

$$sec\frac{11\pi}{6} = sec330° = \frac{1}{cos330°} = \frac{1}{+cos(360° - 330°)} = \frac{1}{cos30°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$cossec\frac{7\pi}{6} = cossec210° = \frac{1}{sen210°} = \frac{1}{-sen(210° - 180°)} = -\frac{1}{sen30°} = -\frac{1}{\frac{1}{2}} = -2$$

$$sec\frac{11\pi}{6} = sec330° = \frac{1}{cos330°} = \frac{1}{+cos(360° - 330°)} = \frac{1}{cos30°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$cotg\frac{4\pi}{3} = cotg240° = \frac{1}{tg240°} = \frac{1}{+tg(240° - 180°)} = \frac{1}{tg60°} = \frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$tg\frac{5\pi}{4} = tg225° = +tg(225° - 180°) = tg45° = 1$$

$$E = -2 + \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3} + 1 \rightarrow E = -1 + \frac{3\sqrt{3}}{3} \rightarrow E = -1 + \sqrt{3}$$

$$d)\ E=\frac{sec1020^\circ+cossec510^\circ+tg945^\circ}{cotg1215^\circ}$$

$$\begin{array}{r|l} 1020^\circ & 360^\circ \\ \hline 300^\circ & 2 \end{array}\qquad \begin{array}{r|l} 510^\circ & 360^\circ \\ \hline 150^\circ & 1 \end{array}\qquad \begin{array}{r|l} 945^\circ & 360^\circ \\ \hline 225^\circ & 2 \end{array}\qquad \begin{array}{r|l} 1215^\circ & 360^\circ \\ \hline 135^\circ & 3 \end{array}$$

$$\mathbf{sec1020^\circ=sec300^\circ=\frac{1}{cos300^\circ}=\frac{1}{+cos(360^\circ-300^\circ)}=\frac{1}{cos60^\circ}=\frac{1}{\frac{1}{2}}=2}$$

$$\mathbf{cossec510^\circ=cossec150^\circ=\frac{1}{sen150^\circ}=\frac{1}{+sen(180^\circ-150^\circ)}=\frac{1}{sen30^\circ}=\frac{1}{\frac{1}{2}}=2}$$

$$\mathbf{tg945^\circ=tg225^\circ=+tg(225^\circ-180^\circ)=+tg45^\circ=1}$$

$$\mathbf{cotg1215^\circ=cotg135^\circ=\frac{1}{tg135^\circ}=\frac{1}{-tg(180^\circ-135^\circ)}=-\frac{1}{tg45^\circ}=-\frac{1}{1}=-1}$$

$$sec1020° = sec300° = \frac{1}{cos300°} = \frac{1}{+cos(360° - 300°)} = \frac{1}{cos60°} = \frac{1}{\frac{1}{2}} = 2$$

$$cossec510° = cossec150° = \frac{1}{sen150°} = \frac{1}{+sen(180° - 150°)} = \frac{1}{sen30°} = \frac{1}{\frac{1}{2}} = 2$$

$$tg945° = tg225° = +tg(225° - 180°) = +tg45° = 1$$

$$cotg1215° = cotg135° = \frac{1}{tg135°} = \frac{1}{-tg(180° - 135°)} = -\frac{1}{tg45°} = -\frac{1}{1} = -1$$

$$E = \frac{sec1020° + cossec510° + tg945°}{cotg1215°} \rightarrow E = \frac{2+2+1}{-1} \rightarrow E = \frac{5}{-1} \rightarrow E = -5$$

$$e)\ E = sec\frac{14\pi}{3} + cossec\frac{31\pi}{6}$$

$$\frac{14\pi}{3} = \frac{14.180°}{3} = 14.60° = 840°$$

840° | 360°
120° | 2

$$\frac{31\pi}{6} = \frac{31.180°}{6} = 31.30° = 930°$$

930° | 360°
210° | 2

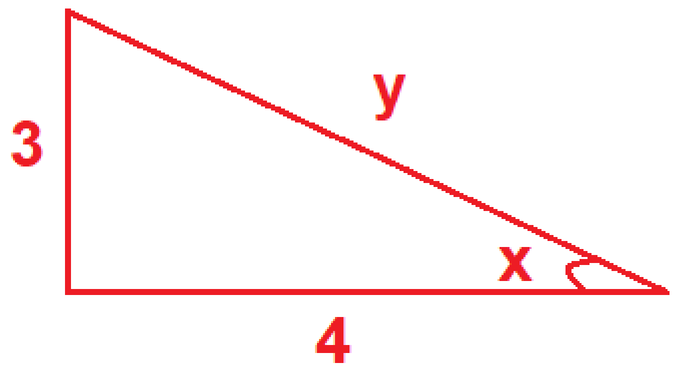
$$sec\frac{14\pi}{3} = sec120° = \frac{1}{cos120°} = \frac{1}{-cos(180° - 120°)} = -\frac{1}{cos60°} = -\frac{1}{\frac{1}{2}} = -2$$

$$cossec\frac{31\pi}{6} = cossec210° = \frac{1}{sen210°} = \frac{1}{-sen(210° - 180°)} = -\frac{1}{sen30°} = -\frac{1}{\frac{1}{2}} = -2$$

$$E = -2 - 2 \rightarrow E = -4$$

3) $Sabendo\ que\ tgx = \frac{3}{4}\ e\ que\ \pi < x < \frac{3\pi}{2}, resolva\ a\ expressão\ E = \frac{secx + cossecx}{cotgx}.$

$$\pi < x < \frac{3\pi}{2} \rightarrow x \in 3^{o}\ quadrante$$



$$y^2 = 3^2 + 4^2 \rightarrow y^2 = 9 + 16 \rightarrow y^2 = 25 \rightarrow y = 5$$

$$cosx = -\frac{4}{5} \rightarrow secx = -\frac{5}{4}$$

$$senx = -\frac{3}{5} \rightarrow cossecx = -\frac{5}{3}$$

$$tgx = \frac{3}{4} \rightarrow cotgx = \frac{4}{3}$$

$$E = \frac{-\frac{5}{4} - \frac{5}{3}}{\frac{4}{3}} \rightarrow E = \frac{\frac{-15 - 20}{12}}{\frac{4}{3}} \rightarrow E = \frac{\frac{-35}{12}}{\frac{4}{3}} \rightarrow E = -\frac{35}{12} . \frac{3}{4} \rightarrow E = -\frac{35}{16}$$

4) *Encontre o valor de m sabendo que* $sen\alpha = \sqrt{m^2 - 1}$ *e* $cos\alpha = m + 1$.

$$sen^2\alpha + cos^2\alpha = 1 \rightarrow \left(\sqrt{m^2 - 1}\right)^2 + (m + 1)^2 = 1 \rightarrow m^2 - 1 + m^2 + 2m + 1 = 1 \rightarrow 2m^2 + 2m - 1 = 0$$

$$m = \frac{-(2) \pm \sqrt{(2)^2 - 4.2.(-1)}}{2.2} \rightarrow m = \frac{-2 \pm \sqrt{12}}{4} \rightarrow m = \frac{-2 \pm 2\sqrt{3}}{4}$$

$$\begin{cases} m_1 = \dfrac{-2 + 2\sqrt{3}}{4} = \dfrac{-1 + \sqrt{3}}{2} \rightarrow \text{não serve, pois nesse caso } cos\alpha > 1 \\ m_2 = \dfrac{-2 - 2\sqrt{3}}{4} = \dfrac{-1 - \sqrt{3}}{2} \rightarrow ok \end{cases}$$

$$S = \left\{\frac{-1 - \sqrt{3}}{2}\right\}$$

5) $Calcule\ m\ de\ modo\ que\ senx = 2m + 1\ e\ cosx = 4m + 1.$

$$sen^2x + cos^2x = 1 \rightarrow (2m+1)^2 + (4m+1)^2 = 1 \rightarrow 4m^2 + 4m + 1 + 16m^2 + 8m + 1 = 1$$

$$20m^2 + 12m + 1 = 0 \rightarrow m = \frac{-12 \pm \sqrt{(12)^2 - 4.20.1}}{2.20} \rightarrow m = \frac{-12 \pm \sqrt{64}}{40} \rightarrow m = \frac{-12 \pm 8}{40}$$

$$\begin{cases} m_1 = \dfrac{-12+8}{40} = -\dfrac{4}{40} = -\dfrac{1}{10} \\ m_2 = \dfrac{-12-8}{40} = -\dfrac{20}{40} = -\dfrac{1}{2} \end{cases}$$

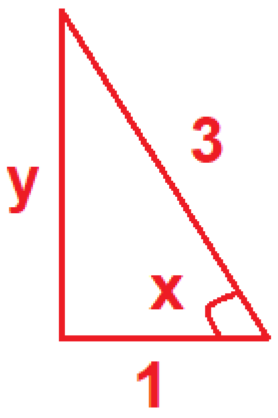
$$m = -\frac{1}{10} \rightarrow senx = -\frac{2}{10} + 1 = \frac{8}{10}\ e\ cosx = -\frac{4}{10} + 1 = \frac{6}{10} \rightarrow ok$$

$$m = -\frac{1}{2} \rightarrow senx = -\frac{2}{2} + 1 = 0\ e\ cosx = 4.-\frac{1}{2} + 1 = -1 \rightarrow ok$$

$$S = \left\{-\frac{1}{10}; -\frac{1}{2}\right\}$$

6) $Sabendo\ que\ secx = 3\ e\ que\ x\ está\ no\ quarto\ quadrante, calcule\ tgx + cotgx.$

$$x \in 4° \ quadrante\ e\ secx = 3 \rightarrow cosx = \frac{1}{3}$$



$$3^2 = 1^2 + y^2 \rightarrow 9 = 1 + y^2 \rightarrow y^2 = 8 \rightarrow y = \sqrt{8} \rightarrow y = 2\sqrt{2}$$

$$tgx = -\frac{2\sqrt{2}}{1} \rightarrow tgx = -2\sqrt{2}$$

$$cotgtx = -\frac{1}{2\sqrt{2}} . \frac{\sqrt{2}}{\sqrt{2}} \rightarrow cotgx = -\frac{\sqrt{2}}{4}$$

$$tgx + cotgx = -2\sqrt{2} - \frac{\sqrt{2}}{4} = -\frac{8\sqrt{2}}{4} - \frac{\sqrt{2}}{4} = -\frac{9\sqrt{2}}{4}$$

7) O gráfico a seguir representa a função:

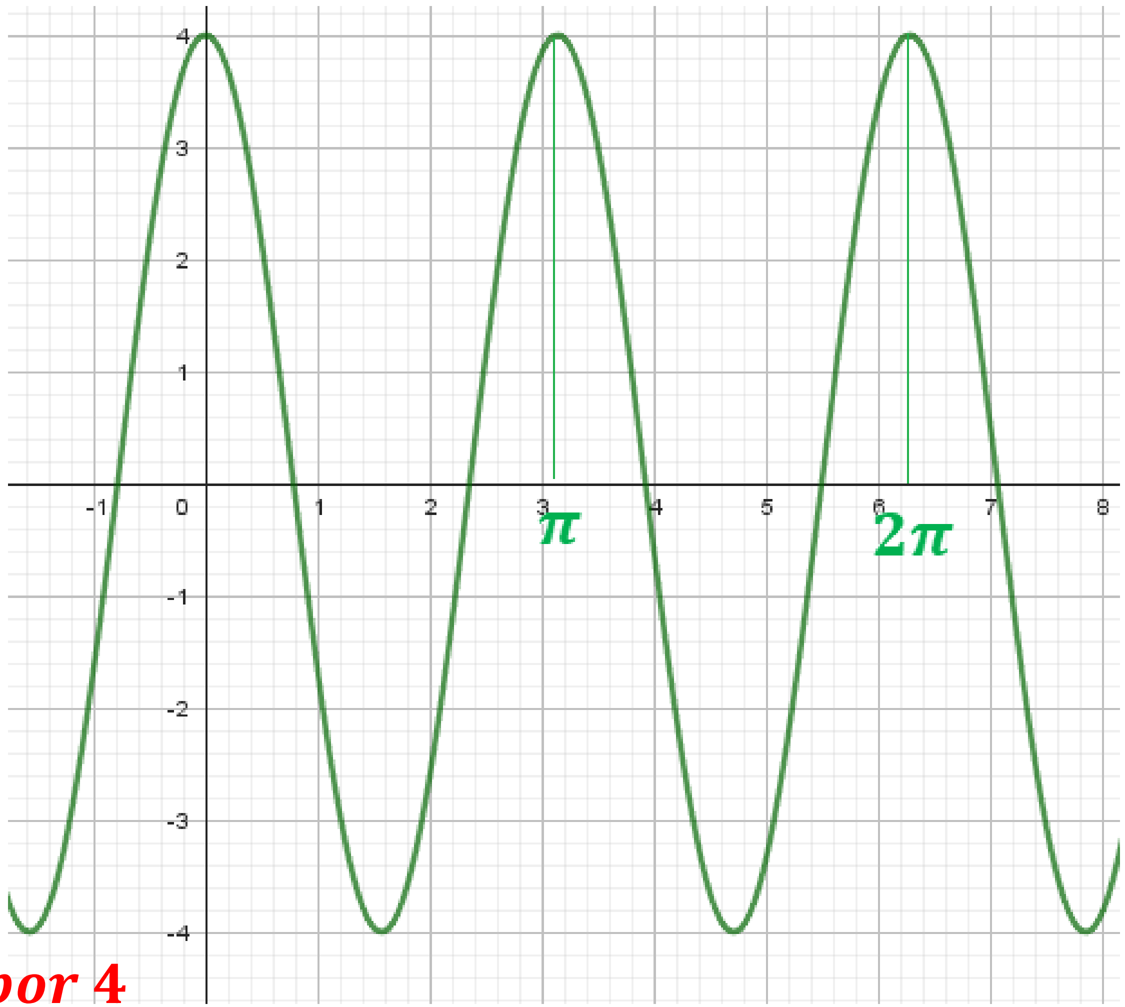
$a)\ y = 4.sen(2x)$

$b)\ y = 2.sen(2x)$

$c)\ y = 4.\cos(2x)$

d) $y = \cos(x)$

e) $y = 4.cosx$



$$p = \pi\ e\ p = \frac{2\pi}{k} \rightarrow \pi = \frac{2\pi}{k} \rightarrow k = 2$$

$Im = [-4, 4] \rightarrow A\ função\ estará\ multiplicada\ por\ 4$

$Como\ o\ gráfico\ não\ começa\ na\ origem, a\ função\ é\ cosseno.$

$$f(x) = 4.cos(2x)$$

$GABARITO: C$

8) O gráfico a seguir representa a função:

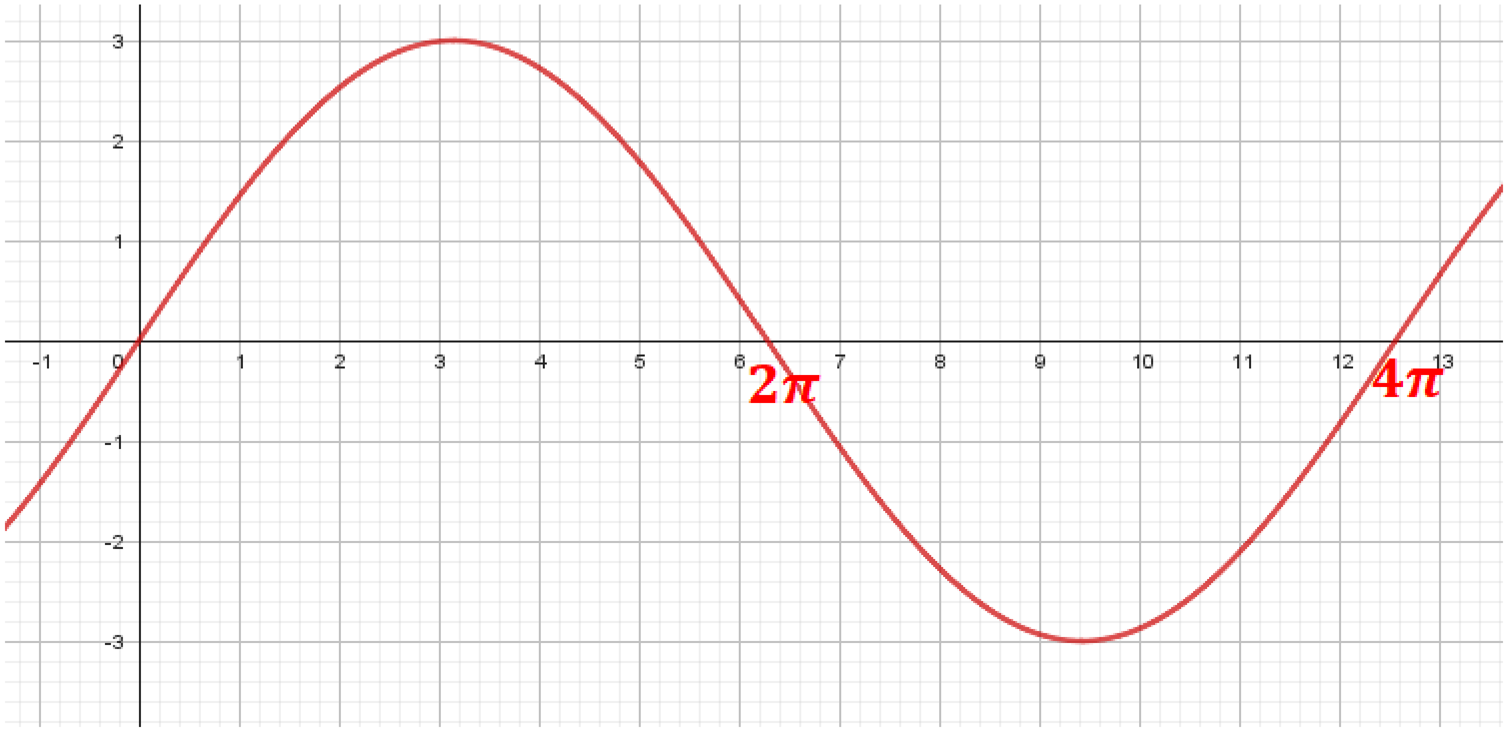
a) $y = 3.sen(2x)$

b) $y = 3.sen\left(\frac{x}{2}\right)$

c) $y = 3.\cos(2x)$

d) $y = 3.\cos\left(\frac{x}{2}\right)$

e) $y = sen\left(\frac{x}{2}\right)$



$\boldsymbol{Im = [-3, 3]}$

$\boldsymbol{p = 4\pi \ e \ p = \frac{2\pi}{k} \rightarrow 4\pi = \frac{2\pi}{k} \rightarrow k = \frac{1}{2}}$

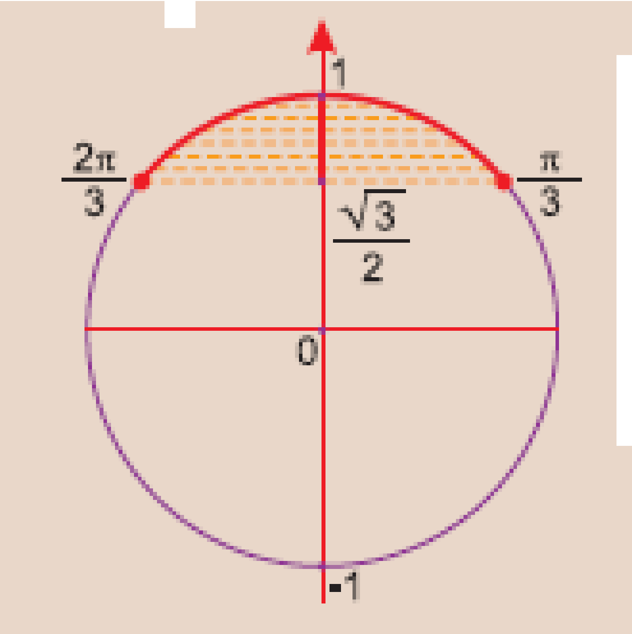
$\boldsymbol{O\ gráfico\ passa\ pela\ origem.\ Função\ seno.}$

$\boldsymbol{f(x) = 3.sen\left(\frac{x}{2}\right)}$

$\boldsymbol{GABARITO: B}$

9) *Resolva a inequação* $2.senx - \sqrt{3} \geq 0$, *sendo* $x \in [0, 2\pi]$.

$$2.senx - \sqrt{3} \geq 0 \rightarrow 2senx \geq \sqrt{3} \rightarrow senx \geq \frac{\sqrt{3}}{2}$$



$$senx = \frac{\sqrt{3}}{2} \rightarrow \begin{cases} x = 60° = \dfrac{\pi}{3} \\ x = 120° = \dfrac{2\pi}{3} \end{cases}$$

$$S = \left\{ x \in R \middle| \frac{\pi}{3} \leq x \leq \frac{2\pi}{3} \right\}$$

10) Resolva as equações abaixo:

$a)\ 2.cosx + 1 = 0, com\ 0° < x < 360°$

$$\mathbf{2.cosx + 1 = 0 \rightarrow 2.cosx = -1 \rightarrow cosx = -\frac{1}{2}}$$

$$\mathbf{cosx < 0 \rightarrow x \in 2°\ quadrante\ ou\ x \in 3°\ quadrante}$$

$$\mathbf{No\ 1^{o}\ quadrante, cosx = \frac{1}{2} \rightarrow x = 60°}$$

$$\begin{cases} \mathbf{x \in 2°\ quadrante \rightarrow x = 180° - 60° \rightarrow x = 120°} \\ \mathbf{x \in 3°\ quadrante \rightarrow x = 180° + 60° \rightarrow x = 240°} \end{cases}$$

$$\mathbf{S = \{120°, 240°\}}$$

$b)\ 2.cos^2x = 1 - senx, 0° < x < 360°$

$$\boldsymbol{sen^2x + cos^2x = 1 \rightarrow cos^2x = 1 - sen^2x}$$

$$\boldsymbol{2.(1 - sen^2x) = 1 - senx \rightarrow 2 - 2.sen^2x = 1 - senx \rightarrow 2.sen^2x - senx - 1 = 0}$$

$$\boldsymbol{Seja\ senx = t \rightarrow 2.t^2 - t - 1 = 0 \rightarrow \Delta = (-1)^2 - 4.2.(-1) \rightarrow \Delta = 9}$$

$$\boldsymbol{t = \frac{-(-1) \pm \sqrt{9}}{2.2} \rightarrow t = \frac{1 \pm 3}{4} \rightarrow \begin{cases} t_1 = \frac{1+3}{4} = 1 \\ t_2 = \frac{1-3}{4} = -\frac{1}{2} \end{cases}}$$

$$senx = 1 \rightarrow x = 90°$$

$$senx = -\frac{1}{2} \rightarrow x \in 3° \; quadrante \; ou \; x \in 4° \; quadrante$$

$$No \; 1° \; quadrante, senx = \frac{1}{2} \rightarrow x = 30°$$

$$\begin{cases} x \in 3° \; quadrante \rightarrow x = 180° + 30° \rightarrow x = 210° \\ x \in 4° \; quadrante \rightarrow x = 360° - 30° \rightarrow x = 330° \end{cases}$$

$$S = \{90°, 210°, 330°\}$$

$c)\ tgx = 1$

$$\boldsymbol{tgx > 0 \rightarrow x \in 1° \ quadrante \ ou \ x \in 3° \ quadrante}$$

$$\boldsymbol{x = 45° + 360°.k, k \in Z}$$

$$\boldsymbol{x = 180° + 45° \rightarrow x = 225° + 360°.k, k \in Z}$$

$$\boldsymbol{S = \{45° + 360°.k, 225° + 360°.k, k \in Z\}}$$

11) *Qual o valor máximo da expressão* $E = \dfrac{3}{\cos\alpha - 2}$?

$$-1 \leq \cos\alpha \leq 1$$

$$\cos\alpha = -1 \rightarrow E = \frac{3}{-1-2} \rightarrow E = \frac{3}{-3} \rightarrow E = -1$$

$$\cos\alpha = 1 \rightarrow E = \frac{3}{1-2} \rightarrow E = \frac{3}{-1} \rightarrow E = -3$$

$$\text{Valor máximo} = -1$$

12) *Sabendo que* $\cos(2x) = \frac{1}{2}$ *e* $0 < x < \frac{\pi}{2}$, *calcule*:

a) *senx*; *b*) *cosx*; *c*) *tgx*

$$a)\ \cos(2x) = \frac{1}{2} \to \cos^2 x - sen^2 x = \frac{1}{2} \to 1 - sen^2 x - sen^2 x = \frac{1}{2} \to 1 - 2sen^2 x = \frac{1}{2}$$

$$1 - \frac{1}{2} = 2sen^2 x \to 2sen^2 x = \frac{1}{2} \to sen^2 x = \frac{1}{4} \to senx = \pm\frac{1}{2}$$

$$como\ x \in 1^{\underline{o}}\ quadrante \to senx = \frac{1}{2}$$

$$b)\ \cos(2x) = \frac{1}{2} \to \cos^2 x - sen^2 x = \frac{1}{2} \to \cos^2 x - (1 - \cos^2 x) = \frac{1}{2} \to 2\cos^2 x - 1 = \frac{1}{2}$$

$$2\cos^2 x = \frac{1}{2} + 1 \to 2\cos^2 x = \frac{3}{2} \to \cos^2 x = \frac{3}{4} \to cosx = \pm\frac{\sqrt{3}}{2}$$

$$como\ x \in 1^{\circ}\ quadrante \to cosx = \frac{\sqrt{3}}{2}$$

$$c)\ tgx=\frac{senx}{cosx}\rightarrow tgx=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\rightarrow tgx=\frac{1}{2}.\frac{2}{\sqrt{3}}\rightarrow tgx=\frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}}\rightarrow tgx=\frac{\sqrt{3}}{3}$$

13) $Simplifique: \dfrac{sen^2x}{senx.cos^2x+sen^3x}.$

$$\frac{sen^2x}{senx.cos^2x+sen^3x}=\frac{sen^2x}{senx.(cos^2x+sen^2x)}=\frac{senx.senx}{senx.1}=senx$$

14) $A\ função\ f(x) = a + b.sen(3x + \pi)\ tem\ Conjunto\ Imagem\ [-1, 5].\ Determine\ a\ e\ b.$

$$\begin{cases} sen(3x + \pi) = 1 \rightarrow a + b.1 = 5 \\ sen(3x + \pi) = -1 \rightarrow a + b(-1) = -1 \end{cases}$$

$$\begin{cases} a + b = 5 \\ a - b = -1 \end{cases} \rightarrow 2a = 4 \rightarrow a = 2$$

$$2 + b = 5 \rightarrow b = 3$$

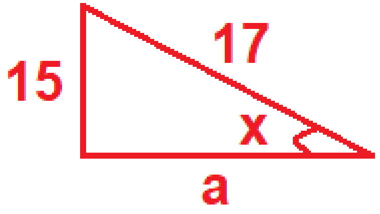
15) $A\ função\ f(x) = 3.cos(kx + 3\pi)\ tem\ período\ p = \frac{3\pi}{2}.\ Determine\ o\ valor\ de\ k.$

$$p = \frac{2\pi}{k} \rightarrow \frac{3\pi}{2} = \frac{2\pi}{k} \rightarrow \frac{3}{2} = \frac{2}{k} \rightarrow 3k = 4 \rightarrow k = \frac{4}{3}$$

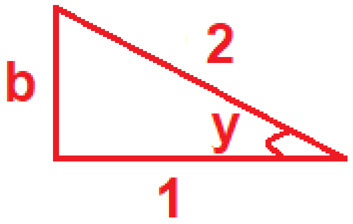
16) $Sabendo\ que\ senx = \frac{15}{17}, cosy = \frac{1}{2}, x\ está\ no\ 2°\ quadrante\ e\ y\ está\ no\ 4°\ quadrante, determine:$

$a)\ sen(x-y); b)\ cos(x-y)\ ; c)\ tg(2y)\ ; d)\ cos(2y)$

$$17^2 = 15^2 + a^2 \rightarrow 289 = 225 + a^2 \rightarrow 64 = a^2 \rightarrow a = 8$$



$$\begin{cases} senx = \frac{15}{17} \\ cosx = -\frac{8}{17} \\ tgx = -\frac{15}{8} \end{cases}$$



$$2^2 = 1^2 + b^2 \rightarrow 4 = 1 + b^2 \rightarrow b^2 = 3 \rightarrow b = \sqrt{3}$$

$$\begin{cases} seny = -\frac{\sqrt{3}}{2} \\ cosy = \frac{1}{2} \\ tgy = -\sqrt{3} \end{cases}$$

$a)\ sen(x-y);\ b)\ cos(x-y)\ ;\ c)\ tg(2y)\ ;\ d)\ cos(2y)$

$$\begin{cases} senx = \dfrac{15}{17} \\ cosx = -\dfrac{8}{17} \\ tgx = -\dfrac{15}{8} \end{cases} \qquad \begin{cases} seny = -\dfrac{\sqrt{3}}{2} \\ cosy = \dfrac{1}{2} \\ tgy = -\sqrt{3} \end{cases}$$

$$a)\ sen(x-y) = senx.cosy - seny.cosx = \frac{15}{17}.\frac{1}{2} - \left(-\frac{\sqrt{3}}{2}\right).\left(-\frac{8}{17}\right) = \frac{15}{34} - \frac{8\sqrt{3}}{34} = \frac{15-8\sqrt{3}}{34}$$

$$b)\ cos(x-y) = cosx.cosy + senx.seny = \left(-\frac{8}{17}\right).\frac{1}{2} + \frac{15}{17}.\left(-\frac{\sqrt{3}}{2}\right) = -\frac{8}{34} - \frac{15\sqrt{3}}{34} = \frac{-8-15\sqrt{3}}{34}$$

$$c)\ tg(2y) = \frac{2tgy}{1-tg^2y} = \frac{2.(-\sqrt{3})}{1-(-\sqrt{3})^2} = \frac{-2\sqrt{3}}{1-3} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

$$d)\ cos(2y) = cos^2y - sen^2y = \left(\frac{1}{2}\right)^2 - \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$